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The impact of renormalization group theory on magnetism

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Abstract. The basic issues of renormalization group (RG) theory, i.e. universality, crossover phenomena, relevant interactions etc. are verified experimentally on magnetic materials. Universality is demonstrated on account of the saturation of the magnetic order parameter for $T \to 0$. Universal means that the deviations with respect to saturation at $T = 0$ can perfectly be described by a power function of absolute temperature with an exponent ε that is independent of spin structure and lattice symmetry. Normally the T^{ε} function holds up to $\sim 0.85 T_c$ where crossover to the critical power function occurs. Universality for $T \to 0$ cannot be explained on the basis of the material specific magnon dispersions that are due to atomistic symmetry. Instead, continuous dynamic symmetry has to be assumed. The quasi particles of the continuous symmetry can be described by plane waves and have linear dispersion in all solids. This then explains universality. However, those quasi particles cannot be observed using inelastic neutron scattering. The principle of relevance is demonstrated using the competition between crystal field interaction and exchange interaction as an example. If the ratio of crystal field interaction to exchange interaction is below some threshold value the local crystal field is not relevant under the continuous symmetry of the ordered state and the saturation moment of the free ion is observed for $T \to 0$. Crossover phenomena either between different exponents or between discrete changes of the pre-factor of the T^{ε} function are demonstrated for the spontaneous magnetization and for the heat capacity.

PACS. 05.10.Cc Renormalization group methods – 75.30.Ds Spin waves – 75.40.Cx Static properties (order parameter, static susceptibility, heat capacities, critical exponents, etc.)

1 Introduction

Renormalization group (RG) theory knows a number of important terms such as stable fixed points, continuous dynamic symmetry, universality, relevant and non relevant interactions, crossover phenomena etc. [1]. The aim of this experimental work is to demonstrate that the concepts of RG theory are in complete agreement with experiment and can easily be verified on magnetic materials.

One of the main conclusions of RG theory is that the thermodynamics of quantities such as the spontaneous magnetization, the heat capacity or the susceptibility is governed by symmetries. Two dynamic symmetries have to be distinguished in solids: the atomistic dynamic symmetry at high temperatures and the continuous dynamic symmetry at low temperatures. The material specific, i.e. non universal properties are all on the length scale of the inter-atomic distance and therefore are defined by the atomistic symmetry. On this length scale the discrete translational symmetry elements of the lattice are decisive. In particular, the atomistic structure presets a short wavelength (high energy) limit for phonons and magnons. Saturation of the heat capacity towards the Dulong-Petit asymptotic value as well as the Curie-Weiss susceptibility are essentially a consequence of the discrete lattice structure and the finite near neighbour interaction energies. On this length scale the crystal electric field and the short range Heisenberg interactions are the relevant interactions in the magnetic materials.

On approaching the critical point the correlation length expands and, finally, becomes larger than the near neighbour distance. This entails dynamic percolation and crossover to continuous dynamic symmetry. The typical symmetry in the vicinity of the critical point is invariance with respect to transformations of the length scale [1]. This is the definition of a continuum. As a consequence, from a dynamic point of view the magnets have to be treated as a continuum. As is well known continuous dynamic symmetry holds in the critical range above and below T_c. This is the justification for the application of field theoretical methods for the calculation of critical exponents [2]. In other words, the crossover from atomistic to continuous dynamic symmetry can be localized at the change from Curie-Weiss susceptibility to critical susceptibility.

Continuous dynamic symmetry holds not only for $T \to T_c$ but also for $T \to 0$. $T = 0$ is another stable fixed

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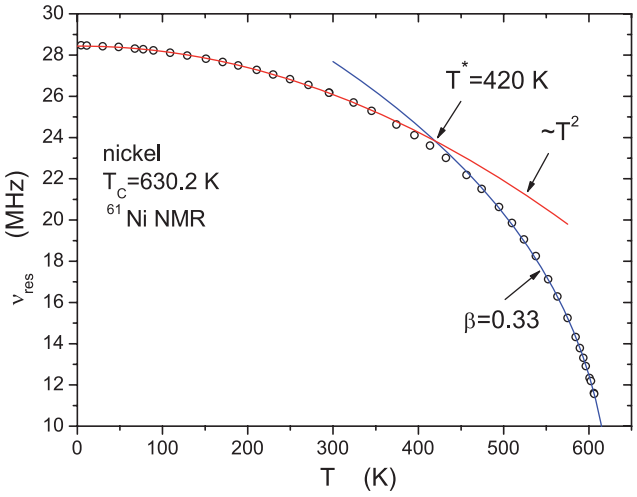

Fig. 1. Spontaneous magnetization of fcc nickel measured by zero-field ^{61}Ni NMR as a function of absolute temperature [12]. Crossover from universal T^2 power function for $T \to 0$ to universal critical power function with $\beta \sim 1/3$ for $T \to T_c$ is at $T^* = 420$ K. The direct crossover between the two universal power functions shows that universality holds in the whole ordered state. The T^2 universality class is characteristic for isotropic 3D interactions and half-integer spin [5]. It can be assumed that $S_{eff} = 1/2$.

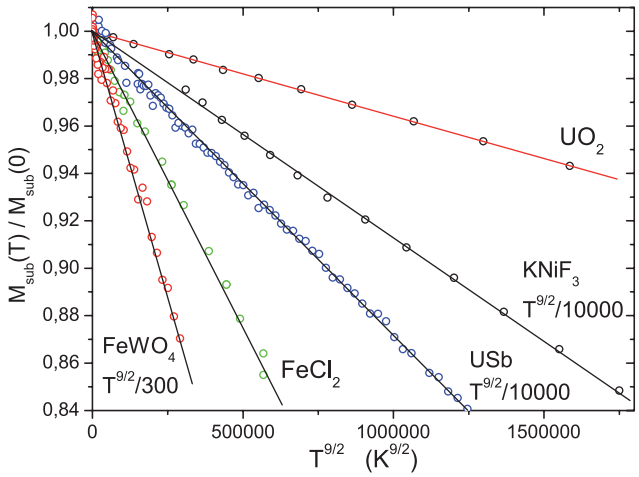

Fig. 2. Normalized sublattice magnetizations for a selection of antiferromagnets with isotropic 3D interactions and integer spins as a function of $T^{9/2}$. For $KNiF_3$ and USb the asymptotic function for $T \to 0$ is T^2. This is not resolved on the $T^{9/2}$ scale. $FeWO_4$ and $FeCl_2$ have $S = 2$, $KNiF_3$ has $S = 1$, UO_2 has $S = 1$ [13,14] and USb has $S = 1$ [8]. For $FeWO_4$, USb and $KNiF_3$ the abscissa values are divided by the given factors.

point. This is well known from Debye's continuum theory of the lattice dynamics of the non magnetic solids. The universal T^3 law predicted for the heat capacity of the non magnetic solids is confirmed for many materials with different chemical compositions and lattice symmetries. In the non magnetic solids the change from continuous to atomistic dynamic symmetry can be localized at the validity limit of Debye's T^3 function [3,4]. This is typically at temperatures between 10 and 30 K.

Also the magnetic order parameter shows universality for $T \to 0$ [5]. This means that the deviations with respect to saturation at $T = 0$ can precisely be described by a single universal power function of absolute temperature (see Figs. 1 and 2). This power function normally holds up to $\sim 0.85 T_c$ where crossover to the critical power function occurs [6]. As a consequence, universality, i.e. continuous dynamic symmetry holds for all temperatures in the long range ordered state and in the critical range above T_c.

The experimentally known universal exponents of the magnetic order parameter for $T \to 0$ are empirical and await a theoretical explanation. They depend, of course, on the dimensionality of the relevant interactions but, surprisingly, also on whether the spin quantum number is integer or half-integer [5,6]. This is equivalent to an odd or even number of states, N, per magnetic particle through $N = 2S + 1$. Note that a spin dependent thermodynamics is observed only in the ordered state but not in the Curie-Weiss regime of the paramagnetic phase [7]. This is indicative of a change of the relevant excitations at the crossover from atomistic to continuous dynamic symmetry. The number of degrees of freedom per spin, i.e. $2S+1$ seems to be the only atomistic detail that remains important for the continuous dynamics.

Continuous dynamic symmetry implies a particular type of excitations. Typical for those excitations is a linear dispersion. This is well known from the electromagnetic radiation in vacuum and from Debye's continuum theory of the lattice dynamics. The vacuum is clearly a continuum with no short wavelength (high energy) limit. Photons and Debye quasi particles can be described by plane waves.

On the other hand, the dispersion relations of the atomistic phonons and magnons are essentially non linear. In particular these dispersions saturate towards a finite energy at the Brillouin zone boundary. We must, hence, assume that the quasi particles of the continuous and atomistic symmetry represent different degrees of freedom with particular dispersions and densities of states. These two quasi particles coexist. According to RG theory we have to ask which one is relevant and defines the dynamics. This will be the quasi particles with the lower dispersion energy or with the higher density of states. In this way the two types of excitations become relevant alternatively. For instance, at low energies the quasi particles of the Debye theory have very similar dispersions as the acoustic phonons but they have higher density of states and therefore are relevant. Beyond the universal T^3 function of the heat capacity the phonons are the relevant excitations. The change from one to the other type of excitations gives rise to a crossover in the heat capacity [3,4].

The magnon excitation spectra evaluated by inelastic neutron scattering exhibit no obvious universal feature. In particular, for ferromagnets the magnon dispersion curves start essentially as a quadratic function of wave vector but for antiferromagnets as a linear function of wave vector. Exceptions from this rule can occur and have been

observed for the antiferromagnet CeAs for which nearly quadratic magnon dispersion has been observed [8]. For the ferromagnet EuS nearly linear magnon dispersion was observed at very low wave vector values [9]. In other words there is neither systematic nor universality in the magnon dispersion relations. As a consequence, the magnons cannot be responsible for the observed universal power functions of temperature. The magnons are responsible for the material specific transition temperature. Once the transition temperature is established the quasi particles of the continuous symmetry govern the temperature dependence.

To summarize, the observed universality cannot be explained by atomistic models but requests a continuum theory of magnetism. In a continuum there are no interactions but only excitations. The quasi particles due to the continuous dynamic symmetry are plane waves with linear dispersion in all solids. This then explains universality. On the other hand, the quasi particles of the atomistic symmetry, the magnons, are determined by the interactions between the atomistic spins. The general properties of the (quasi)particles due to continuous symmetry have been described by Goldstone, Salam and Weinberg [10]. Similar as photons in vacuum and sound waves in solids these particles have neither mass nor magnetic moment. As a consequence they cannot be observed using inelastic neutron scattering.

2 Universality

The magnetic order parameter shows universal temperature dependence in the vicinity of the stable fixed points $T = 0$ and $T = T_c$ [6]. Universality is represented by a temperature power function of the distance from the stable fixed point. For $T \to T_c$ this is the well known critical power function of the form $(T_c - T)^\beta$. For $T \to 0$ the universal function is a power function of absolute temperature, T^ε. Crossover between the power function for $T \to 0$ and for $T \to T_c$ is typically at $T^* \sim 0.85 T_c$ [6]. At this crossover temperature the correlation length has a minimum of typically two times the near neighbour distance [11]. This seems to be sufficient to establish dynamic percolation and continuous dynamic symmetry in the whole ordered range.

Figure 1 visualizes the direct crossover from universal power function for $T \to 0$ to critical power function for $T \to T_C$ for ferromagnetic fcc nickel. These data are obtained by zero field ^{61}Ni NMR [12]. Since nickel is cubic for all temperatures the interactions can be assumed to be 3D isotropic. We have identified the T^2 universality class as characteristic for isotropic 3D magnets with half-integer spin, i.e. with an even number of relevant states [5]. Considering that the saturation magnetic moment is 0.617 μ_B per Ni it appears reasonable to assume that the effective spin is $S_{eff} = 1/2$, i.e. that there are two relevant states. The crossover between the two power functions is at $T^* = 420$ K. The critical range, therefore, is unusually large. The critical exponent is close to $\beta \sim 1/3$.

Table 1. Spin quantum numbers, S, Néel temperatures, T_N and magnon excitation gaps, E_{gap}, for a selection of antiferromagnets with integer spin and isotropic 3D interactions ($T^{9/2}$ universality class). The origin of the gap is not clear in these isotropic magnets. Because of a different temperature dependence of gap and sublattice magnetization the gap is considered as an independent order parameter. The fact that E_{gap} is not proportional to T_N supports this. Sublattice magnetization and gap decrease by $T^{9/2}$ functions.

	UO_2	$FeCl_2$	USb	$KNiF_3$	YVO_3
	$S = 1$	$S = 2$	$S = 1$	$S = 1$	$S = 1$
T_N (K)	30.8	23.55	217	245.5	116.1
E_{gap}/k_B (K)	26	24	74	70	80
Reference	[15]	[16]	[8]	[17]	[18]

Figure 2 demonstrates universality of the order parameter for $T \to 0$ for a selection of antiferromagnets with integer spin and isotropic 3D interactions. These data are obtained partly on powder materials ($FeCl_2$, $FeWO_4$) but partly on single crystals (UO_2, $KNiF_3$, USb) using neutron scattering (instruments E6 and E1 of HMI/Berlin). $FeCl_2$ and $FeWO_4$ have $S = 2$ while $KNiF_3$ has $S = 1$. USb has $S = 1$ [8]. UO_2 also has $S = 1$ [13,14].

Linear dependence on $T^{9/2}$ temperature scale for all samples shows that materials with different chemical compositions and lattice symmetries can fall in the same universality class. The material specific properties enter the pre-factor of the $T^{9/2}$ function only and are determined essentially by the (non universal) magnetic ordering temperature, i.e. by the magnetic hardness. We have identified the $T^{9/2}$ universality class as characteristic for isotropic 3D magnets with integer spin [5]. Note that isotropic 3D behaviour can occur also in non cubic materials. Condition for this is that the anisotropy of the interactions is below some threshold value. The anisotropy of the interactions is then not relevant, i.e. it does not induce a crossover into a lower symmetry class.

We should note that in $KNiF_3$ and USb the $T^{9/2}$ function is not the asymptotic behaviour for $T \to 0$ [7]. Asymptotically T^2 function is observed. This detail is not resolved in Figure 2. Normalization of the $T^{9/2}$ function to unity for $T \to 0$ therefore slightly underestimates the saturation value.

As far as experimental information is available there seems to be always a magnon excitation gap in the magnets of the isotropic 3D universality class with integer spin [7]. If the magnons would be relevant one would expect that the order parameter decreases by an exponential function. The observed power functions confirm that the magnons are not relevant and that the relevant excitation spectrum must be continuous.

The gap decreases as the order parameter by $T^{9/2}$ function but the pre-factor of the $T^{9/2}$ function is much larger, typically by a factor of 2.5. Because of this different temperature dependence the gap has been identified as a second order parameter. In Table 1 the magnon gap values and the transition temperatures of some materials

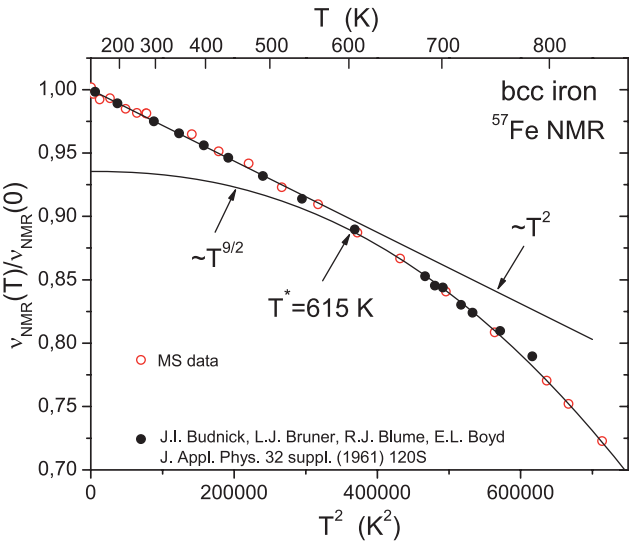

Fig. 3. Zero field spontaneous magnetization of bcc iron measured by ^{57}Fe Mössbauer spectroscopy (MS) [19] and ^{57}Fe NMR [20] vs. T^2. At $T^* = 615$ K crossover from T^2 function to $T^{9/2}$ function can be identified. This crossover indicates change from half-integer to integer spin. It can be assumed that this crossover is from $S = 3/2$ to $S = 2$, i.e. from 4 to 5 relevant states.

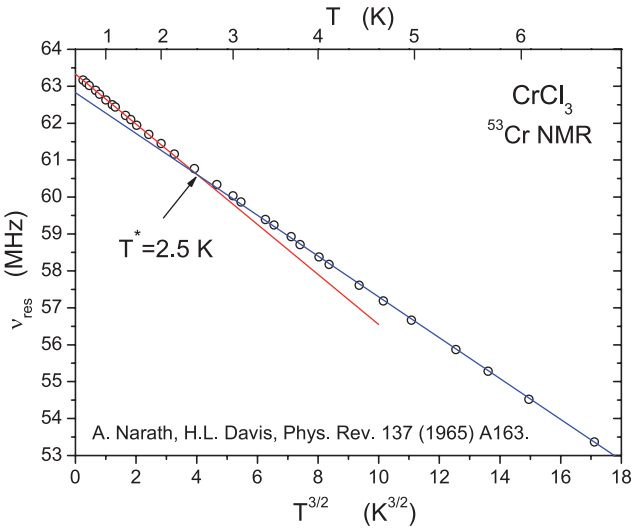

Fig. 4. Crossover between two $T^{3/2}$ functions with different pre-factors (amplitude crossover) in the temperature dependence of the zero field ^{53}Cr NMR (∼order parameter) of $CrCl_3$ after reference [22]. A change of pre-factor indicates a change of a non relevant parameter such as the absolute value of the interactions. Crossover to larger slope at low temperatures means decreasing interactions.

that belong to the $T^{9/2}$ universality class are compiled. It can be seen that the two quantities do not scale.

3 Crossover phenomena

We have to distinguish between different types of crossover events. Figure 1 has demonstrated the crossover of the order parameter between the two stable fixed points $T = 0$ and $T = T_C$ using nickel as an example. This crossover occurs in all magnets [6]. Figure 1 represents the simplest situation with no further crossover event.

Another type of crossover can occur additionally if the number of relevant states changes as a function of temperature. This crossover therefore is characterized by a change of the exponent ε. It can be anticipated that the number of relevant states can only increase with increasing temperature.

Such a crossover seems to occur in the itinerant ferromagnet bcc iron. Since iron has cubic bcc structure for all temperatures the interactions are 3D isotropic throughout. Observation of a crossover therefore can be attributed to a change of the relevant states.

Figure 3 shows normalized ^{57}Fe Mössbauer spectroscopy (MS) data [19] and normalized NMR data [20] as a function of T^2. At low temperatures T^2 function is observed. This is typical for half-integer spin (even number of states). As we have shown before [5] this T^2 function is excellently resolved by the ^{57}Fe NMR data of reference [21]. Considering that the saturation moment of iron is 2.217 μ_B per Fe atom it appears reasonable to assume that the effective spin is $S_{eff} = 3/2$, i.e. the number of relevant states is $N = 4$.

At crossover temperature $T^* = 615$ K the exponent changes from $\varepsilon = 2$ to $\varepsilon = 9/2$. This indicates a change to integer spin. It is reasonable to assume that the effective spin now is $S_{eff} = 2$, i.e. that the number of relevant states has increased from 4 to 5. We should note that significance of a spin quantum number in a magnetic continuum is not completely clear. At the moment we can only assume that the magnetic plane waves can have different polarizations and that these polarizations are related to the atomic spin.

A further type of crossover occurs if the symmetry changes. A crossover of this type is at the change from Curie-Weiss susceptibility to critical susceptibility in the paramagnetic phase. At this crossover the dynamic symmetry changes from atomistic to continuous. Examples for this we have presented in reference [11].

Another very interesting type of crossover we have called amplitude crossover [5]. At this crossover the pre-factor of the T^ε function changes discontinuously. This crossover demonstrates that the pre-factor of the universal power function assumes discrete values only. This is a necessary condition for the universal power function to hold exactly between two crossover events. As an example we show in Figure 4 ^{53}Cr NMR data for hexagonal $CrCl_3$ after reference [22]. The observed $T^{3/2}$ universality class is characteristic for anisotropic 3D interactions and half-integer spin ($S = 3/2$). Since the change in slope is not very large a high experimental accuracy is necessary to resolve an amplitude crossover.

Discrete change of the pre-factor indicates a significant change of a non relevant parameter such as the absolute strength of the magnetic interactions. The crossover to steeper temperature dependence at low temperature can qualitatively be explained by decreasing interactions as

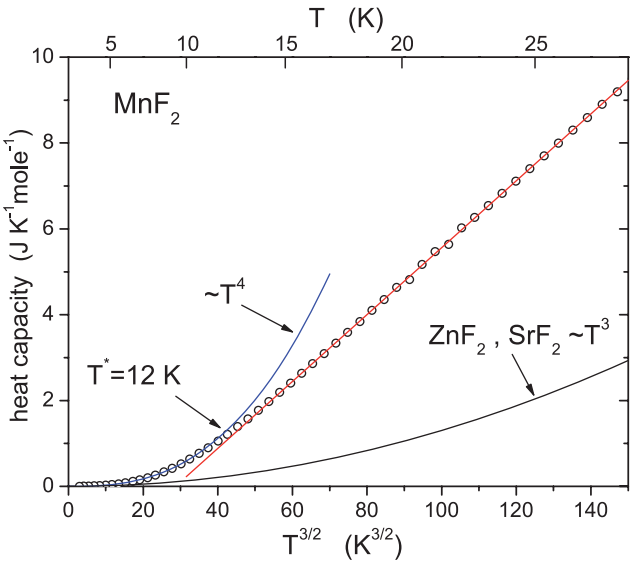

Fig. 5. Molar heat capacity of tetragonal MnF_2 as a function of $T^{3/2}$. Crossover from low temperature T^4 function (fitted exponent 4.036 ± 0.020) to high temperature $T^{3/2}$ function at $T^* = 12$ K is identified. Comparison with the T^3 functions of the non magnetic reference compounds SrF_2 and ZnF_2 [24] shows that the heat capacity of MnF_2 is considerably larger due to the magnetic degrees of freedom. The lattice, therefore, is not relevant and the observed T^4 and $T^{3/2}$ universal functions can be assumed to be defined by the relevant magnetic subsystem.

a function of decreasing temperature. Although the interactions can be assumed to change continuously with temperature a discrete crossover event can eventually be induced by a change beyond some threshold. This demonstrates the stability of the universality classes. Amplitude crossovers we have identified in the spontaneous magnetization of ferromagnetic GdZn [23] in yttrium iron garnet (YIG), $CrBr_3$, ZrF_2, $FeBO_3$ and $YFe_{10}Mo_2$ [5].

The low temperature heat capacity is particularly rich in crossover phenomena [4]. In Figure 5 we show heat capacity data of MnF_2 on $T^{3/2}$ temperature scale. Crossover from low temperature T^4 function to high temperature $T^{3/2}$ function at $T^* = 12$ K is well resolved. These data we have obtained on a single crystal using a Quantum Design PPMS system.

Fitting a power function to the heat capacity data of the range $2 \leqslant T \leqslant 10$ K gives exponent 4.036 ± 0.020. In this temperature range the heat capacity increases by a factor of 500. Comparison with the Debye T^3 functions of the non magnetic reference compounds ZnF_2 and SrF_2 [24] shows that the heat capacity of MnF_2 is considerably larger. This is clearly due to the magnetic degrees of freedom.

It is important to note that universality of lattice and magnetic heat capacity for $T \to 0$ has the consequence that the two ordered subsystems give rise to one common universal power function for $T \to 0$ [4]. The subsystem with the larger heat capacity will be relevant and defines the universal exponent. The subsystem with the smaller heat capacity modifies the pre-factor of the power function only. For MnF_2 with a relatively low Néel temperature of 67.5 K [25,26] the magnetic heat capacity is larger than the lattice heat capacity for $T \to 0$. It can therefore be assumed that the observed T^4 function is essentially defined by the magnetic degrees of freedom. Otherwise Debye's T^3 function would be observed [4]. The T^4 function can, however, occur also in non magnetic materials such as silicon, germanium and in vitreous silica [4].

The exponent of the non asymptotic $T^{3/2}$ function is not observed in non magnetic materials and can clearly be attributed to the magnetic subsystem. Typical for a non asymptotic power function is that it does not extrapolate into the origin. It is therefore important to always include an absolute constant in fitting a power function to the experimental data.

The two exponents 4 and 3/2 are theoretically unexplained at present. We can only assume that they result by the complicated interplay between lattice and magnetic heat capacity with a dominating magnetic contribution. Indication for this is the crossover at $T^* = 12$ K. This temperature is typical for the validity limit of Debye's T^3 function in the non magnetic solids. This crossover therefore could be due to the change from continuous to atomistic dynamic symmetry of the lattice. This process occurs, so to say, in the non magnetic background.

4 Relevant and non relevant interactions

The principle of relevance is very important. Figure 5 has demonstrated this for the competition between lattice and magnetic heat capacity for $T \to 0$. It can be assumed that the strongest interaction will be relevant and defines the universal exponent.

A further example for the principle of relevance is the anisotropy of the magnetic interactions. Since we consider bulk magnets with a pure spin moment there is no single particle anisotropy, i.e. the spin can point in any direction. Moreover the lattice symmetry is always 3D. We can assume that in non cubic magnets the interactions are different along the three space directions. Depending on the ratio between these interactions the magnetic material can fall either in the 3D, 2D or 1D universality class. The magnetic dimensionality is determined by the strongest interaction and can directly be recognized by the domain structure. In 3D magnets there are three types of domains with spin orientations along x-, y- and z-axis, respectively. In cubic magnets these domains are equally populated. In materials with axial lattice symmetry that are magnetically 1D only one domain type with spin orientations along the main symmetry axis exists.

In the magnetically 1D bulk magnets finite transverse interactions can be tolerated up to some threshold value. These transverse interactions weakly couple the spins transverse to the main symmetry axis and let the spin structure appear three-dimensional. The transverse correlation length is, however, finite. This has been evidenced in pioneering neutron scattering studies on tetragonal MnF_2 [27,28]. The finite transverse correlation length

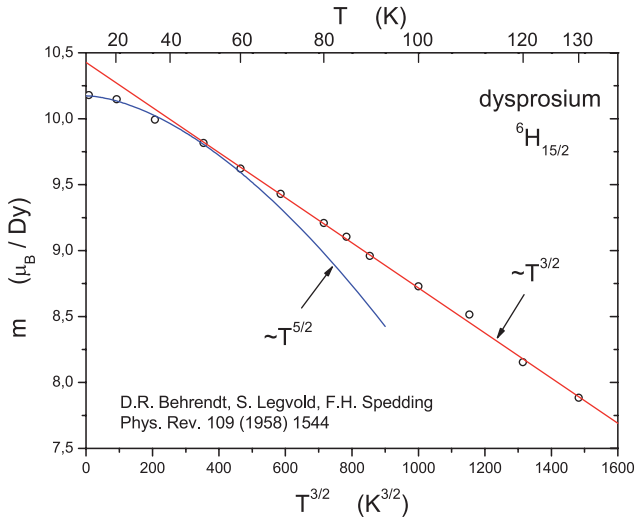

Fig. 6. Spontaneous magnetic moment of dysprosium after reference [32] as a function of $T^{3/2}$. The observed saturation moment of 10.23 μ_B/Dy is in good agreement with 10 μ_B/Dy of the free Dy^{3+} ion and shows that the crystal field interaction is not relevant [29]. The excess moment of 0.23 μ_B/Dy is attributed to polarization contributions of the conduction electrons. $T^{3/2}$ function (anisotropic 3D interactions) is consistent with the half-integer moment $J = 15/2$ of the free Dy^{3+} ion ($^6H_{15/2}$). Crossover to asymptotic $T^{5/2}$ function (1D interaction) is not sufficiently resolved.

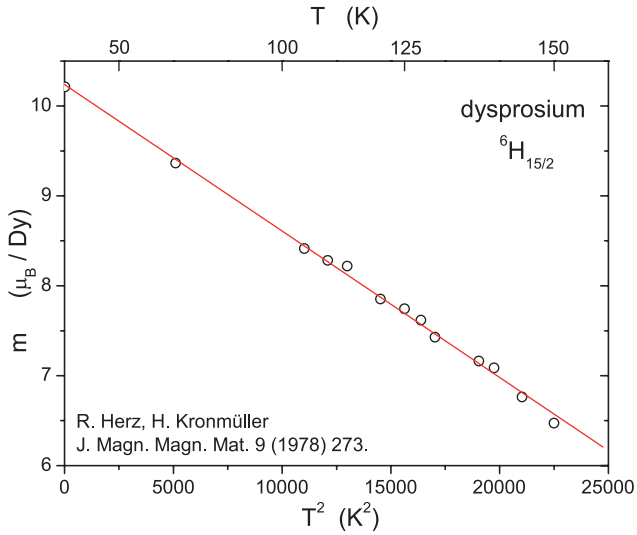

Fig. 7. Spontaneous magnetic moment of dysprosium after reference [34] as a function of T^2. Observation of the full moment of the free Dy^{3+} ion (10 μ_B/Dy) shows that the crystal field is not relevant [29]. The excess moment of 0.24 μ_B/Dy is attributed to polarization contributions of the conduction electrons [30,31]. Observation of T^2 function (isotropic 3D interactions) is in contrast to the result of Figure 6. In spite of a very similar spontaneous magnetization curve the exponents (universality class) can be different due slightly different sample properties.

is defined by the strength of the transverse interactions. These are too weak to be relevant and therefore leave the universality class unchanged, i.e. 1D. As a consequence, the universality classes have a considerable band width. The band width is given by the span of the non relevant interactions.

In magnets with an orbital moment the principle of relevance can nicely be demonstrated on account of the crystal field interaction [29]. The crystal field is a local phenomenon due to the ligands and therefore affects the magnon dispersions at the Brillouin zone boundary. Since the magnon dispersions are not relevant the crystal field splitting is, in a first approximation, also not relevant under the continuous dynamic symmetry of the long range ordered state. This is noticed by a saturation magnetic moment that conforms to the free ion value, i.e. to the absence of the crystal field. Examples for a non relevant crystal field are the ferromagnetic heavy Rare Earth (RE) elements [30,31]. In these materials the crystal field interaction is smaller than the exchange interaction.

A measure for the strength of the crystal field is provided by the anisotropy of the paramagnetic susceptibility. On the other hand, the strength of the exchange interactions can be estimated from the critical temperature. In hexagonal dysprosium, for instance, the crystal field gives rise to a paramagnetic anisotropy of $\Theta^{\perp} - \Theta^{\parallel} = 169 - 121 = 48$ K [30–32]. This is by no means negligible compared to the ordering temperature of 178.5 K. Nevertheless, the observed saturation moment per Dy atom of 10.23 μ_B is even larger than the free Dy^{3+} value of 10 μ_B due to polarization contributions of the conduction electrons (see Fig. 6). A similar situation holds for the other heavy RE elements [30,31].

Observation of the full saturation magnetic moment implies that all $2J + 1$ states of the free Dy^{3+} ion with the electronic configuration $^6H_{15/2}$ are relevant. $J = 15/2$ means $N = 2J + 1 = 16$, i.e. an even number of states. Considering that the lattice symmetry is hexagonal the relevant interactions can be expected to be either 3D anisotropic or 1D. For an even number of states the associated universality classes are $T^{3/2}$ and $T^{5/2}$, respectively [5,6]. These power functions are, in fact, observed. Figure 6 shows the spontaneous magnetization of Dy after reference [32] as a function of $T^{3/2}$. The $T^{3/2}$ dependence is clearly identified. Crossover to low temperature $T^{5/2}$ function is not sufficiently resolved. $T^{5/2}$ function means 1D symmetry. This would be consistent with the expectation that the symmetry of the universality class can only decrease with decreasing temperature. The same type of crossover from $T^{3/2}$ to $T^{5/2}$ is observed also for ferromagnetic Gd with half-integer spin ($S = 7/2$) [33].

We should note that the universality classes can depend sensitively on weak parameters such as the stoichiometry or the lattice strain of the sample. Metastability of the universality classes is illustrated by a comparison between Figures 6 and 7. In contrast to the results of Figure 6 pure T^2 function is identified in the spontaneous magnetization of Dy after reference [34] in Figure 7. T^2 function indicates isotropic 3D behaviour.

The saturation moment of 10.24 μ_B/Dy in Figure 7 [34] is in perfect agreement with 10.23 μ_B/Dy in

Table 2. Relevant crystal field interaction in ferromagnetic heavy rare earth (RE) alloys with composition $REPt_2$. The first column gives the number of states of the free RE^{3+} ion. The second column gives the theoretical saturation moments of the free RE^{3+} ion in Bohr magnetons. The experimental saturation moments, m_{exp}, are after reference [35] (third column). Choosing a suitable effective number of states with $N_{eff} < N_0$ effective saturation moments, m_{eff}, can be calculated according to equation (1) that agree excellently with the observed moments m_{exp}.

	$N_0 = 2J + 1$	m_{theor} (μ_B)	m_{exp} (μ_B)	N_{eff}	m_{eff} (μ_B)
$TbPt_2$	13	9	6.87	10	6.92
$DyPt_2$	16	10	6.87	11	6.87
$HoPt_2$	17	10	8.26	14	8.23
$ErPt_2$	16	9	7.37	13	7.31

Figure 6 [32]. Also the critical temperatures are, practically, identical. In other words, the two material specific ending points of the spontaneous magnetization curve are unchanged. These ending points are determined by the strong atomistic interactions. The universal temperature function(s) between these ending points is due to the excitations of the continuous symmetry and can depend sensitively on various weak parameters. This demonstrates that the excitations of the continuous symmetry are independent of the atomistic interactions and define the magnetic dimensionality.

The effect of a strong crystal field is to reduce the number of relevant states at low temperatures. This gives rise to a saturation moment that is reduced with respect to the free-ion value. It can be assumed that the number of relevant states is always an integer or, equivalently, that the material always fits one of the universality classes. As a consequence, reduction of the observed saturation moment as a function of an increasing crystal field can be expected to be in discrete steps. Any change of the number of states changes the universality class. This was already demonstrated by Figure 3.

The crystal field interaction must be comparable or larger than the exchange interaction in order to become relevant. This seems to be the case for RE compounds or alloys with a relatively low ordering temperature. In Table 2 we have compiled some data for the heavy $REPt_2$ alloys after reference [35]. These alloys have Curie temperatures below 40 K only. If we assume that the crystal field interaction has the same order of magnitude as in the heavy Rare Earth elements the ratio of crystal field interaction to exchange interaction is much larger for the $REPt_2$ alloys than for the RE elements. The crystal field therefore will be relevant.

It can be seen in Table 2 that the experimentally observed saturation moments (m_{exp}) are considerably smaller than the theoretical values (m_{theor}) of the free RE^{3+} ions. The simplest possible explanation is to attribute the reduced moments to an effective number of states N_{eff} that is smaller compared to the total number of states, N_0. In other words we assume

$$m_{exp}/m_{theor} = N_{eff}/N_0. \quad (1)$$

Choosing suitable values for N_{eff} it is possible to calculate m_{eff} values that are in surprising good agreement with m_{exp}. The agreement between m_{exp} and m_{eff} is, in fact, excellent considering that equation (1) assumes a constant Landé factor, independent of the number of relevant states.

5 Conclusions

In the magnetically long range ordered state and in the critical range above T_c the atomistic structure is of no importance for the dynamics. This we conclude from the observed universality in the temperature dependence of the magnetic order parameter for $T \to 0$ and from the well known universality of the critical power functions for $T \to T_c$. Other quantities such as the heat capacity also show universality for $T \to 0$ [4]. It seems that also magnetostriction shows universal temperature dependence [36]. Universality can only be explained in the framework of a continuum theory. However, a continuum theory of magnetism for $T \to 0$ in analogy to Debye's continuum theory of the lattice dynamics is missing.

On the other hand, the atomistic structure and the discrete Heisenberg near neighbour exchange interactions are present also in the ordered state. The experimentally known magnon dispersions prove this. The magnon dispersions are determined by the atomistic symmetry and are material specific. They represent the interactions between the spins and determine (static) properties such as spin structure and transition temperature. In particular, ferromagnets and antiferromagnets have different magnon dispersions. The observed universal exponents are, however, independent of spin structure and lattice symmetry. As a conclusion, the magnons cannot be the relevant excitations.

The observed universal power functions are due to the excitations of the continuous magnet. In a continuum there are no interactions but only excitations. This is a completely different situation compared to the interatomic interactions of the atomistic symmetry. It can be assumed that the quasi particles of the continuous symmetry have linear dispersion in all 3D magnets. In this way universality can be explained. The non universal pre-factor of this linear dispersion (the stiffness constant) scales with the non universal transition temperature and is determined by the atomistic interactions. In other words, atomistic and continuous excitations have clearly distinguished functions. Compared to the magnons the quasi particles of the continuous symmetry must have either lower dispersion energy or higher density of states

in order to be relevant. In this respect the magnons can be viewed as epithermal excitations. The quasi particles of the magnetic continuum have properties as described by Goldstone, Salam and Weinberg, i.e. they have neither magnetic moment nor mass and therefore are not interacting with each other and also not interacting with neutrons [10]. These particles behave as a gas.

Speaking in terms of the wave picture the quasi particles of the continuous symmetry are plane magnetic waves. It is evident that magnetic plane waves carry no net magnetic moment. This makes the quasi particles non interacting. Speaking in terms of the particle picture the quasi particles of the continuous symmetry must be spin compensated in a similar way as it is known from the Cooper pairs of superconductivity [37].

Wavelength and energy of the magnetic fluctuations are limited by the near neighbour interactions in the paramagnetic phase. On this length scale the short range Heisenberg interactions are relevant. These interactions can be extracted from the observed magnon dispersions. They define the Curie-Weiss susceptibility and the non universal transition temperature.

One interesting possibility to directly observe the excitations of the magnetic continuum seems to be provided by the standing magnetic waves in thin magnetic films. These resonating modes along the film normal have been called standing spin waves [38] which appears misleading. Because the standing magnetic waves can be observed also in amorphous ferromagnets and in partly crystalline permalloy films [39] it is justified to identify the standing waves with the excitations of the magnetic continuum.

Normally films with thickness of a few hundred nanometers are used in these resonance experiments. These films are 3D [40]. This is confirmed by the T^2 temperature dependence of the uniform precession mode (the mode with wave vector $k = 0$) [41]. Note that T^2 dependence is observed also in bulk magnets (see Figs. 1 and 3). However, upon excitation of standing waves with sufficiently large wave vector values, i.e. with sufficiently short wavelengths $T^{3/2}$ dependence is observed [42]. The $T^{3/2}$ dependence has been observed in many resonance experiments and is typical for 2D magnets. As a conclusion, a dimensionality crossover from 3D to 2D is induced upon excitation of standing waves. The modulated layered structure due to excitation of standing waves seems to make the films 2D. This proves again the meta-stability of the universality classes.

The crossover from 3D to 2D occurs at small k values. This can be seen in Figure 8. In these resonance experiments the radio frequency commonly is constant and resonance condition is achieved by adjusting the magnetic field. It can be seen that for large order numbers, n, of the standing waves quadratic dispersion is observed (note that $k \sim n$). For small order numbers a linear dispersion can be identified. This means that in two dimensions the excitations of the magnetic continuum have quadratic dispersion but in three-dimensions they have linear dispersion. More detailed experimental studies seem to be necessary to finally confirm this conclusion.

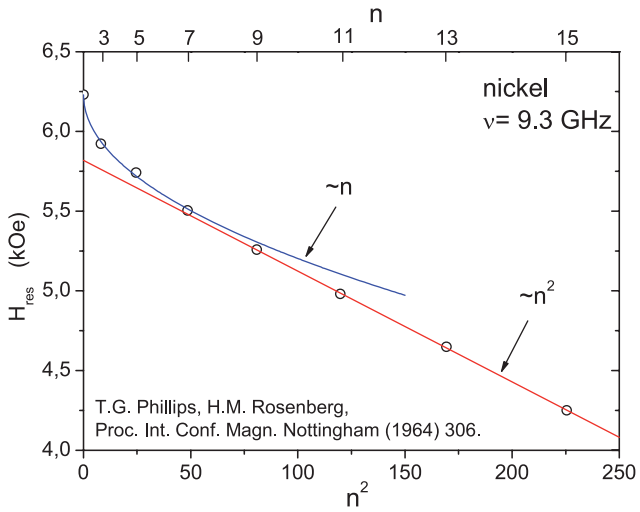

Fig. 8. Field for resonance of standing magnetic waves across a thin nickel film as a function of the order number squared (n^2), after reference [43]. Uniform precession is for $n = 0$. Because of a fixed resonance frequency (9.3 GHz) resonance condition is set by a magnetic field. Crossover from linear dependence $\sim n$ to quadratic dependence $\sim n^2$ can be identified. This means crossover from linear to quadratic dispersion. For $H_{res} \sim n$ the dynamic symmetry of the film is 3D but for $H_{res} \sim n^2$ it is 2D (see text).

The new phenomenon in the long range ordered state is that the dynamics is different for integer and half-integer spin quantum numbers, i.e. for an odd or even number of relevant states [5,6]. This also shows that the universal temperature functions are not due to the Heisenberg interactions. The change from atomistic to continuous interactions is at the crossover from Curie-Weiss susceptibility to critical susceptibility [11]. Because of the spin dependence we can assume that the plane waves of the continuous symmetry have different polarization properties in materials with integer or half-integer spin values.

As a conclusion, a new continuum theory of magnetism has to be developed that explains all features that are outside the scope of the Heisenberg model: universality at $T = 0$, different universality classes for an even and odd number of states per magnetic particle and long range magnetic order in one and two dimensions [44–46].

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